η) the existence postulate for mass-distributions on mass points, implicit in [4, Ftn. 1] but present in [1, p. 151] as Assioma 20, 1 (a)–(b), implies certain causal implications through Def. 1 in [4, Ftn. 1].

Let me add that, in ordinary English, "or" has two different interpretations and "possibility" has an even greater ambiguity; furthermore, asserting (F) practically implies that in Bressan's axiomatic theories, "$\Diamond p$" may be interpreted as "it is logically possible that p". Should this interpretation actually be allowed, I would agree with the presence of "formal counterparts for" in (A).

As for (C) and (D), I regard causal nexus as a very important explicandum in Carnap's sense. It is used in various senses within consideration about physical theories, e.g. in preliminaries or in view of applications. However (likely because of this ambiguity) it is used within no (object) theory of e.g. mathematical physics that I know. Furthermore,

ϑ) my formal axiomatic systems were constructed, sometimes in various versions as in [4] and [5], to render the foundations of the Mach-Painlevé type for some physical theorie rigorous — see sect. 4.

Therefore (D) holds (so far).

3. In [3, pp. 200–201] one considers any sentence asserting that a certain phenomenon occurs in the space-time region $\mathcal{R}_4$; and for $r = 1, 2$ one (intuitively) explains the notions of *causal* and *physical possibility*$_r$: see γ's — where "r" stands for "in the r-th sense". E.g. "p's physical possibility$_2$" briefly means p's compatibility with physical law. Furthermore the following is in effect stated.

A3.1 For $\mathcal{R}_4$ bounded, p is said to be *causally possible*$_1$ if p would be technically possible for suitable skilful technicians,[3] i.e. if some technicians as skilful as some possible future technicians would be able to implement p (by working outside $\mathcal{R}_4$),[4] with an arbitrarily preassigned degree of approximation.

A3.2 For $\mathcal{R}_4$ unbounded, p is said to be *causally possible*$_1$ if so is p's restriction to every bounded part of $\mathcal{R}_4$.

A3.3 Physical sentences that are causally possible$_1$ are said to be *physically possible*$_1$.

[3] E.g. in [3] Bressan uses the natural notion of technical possibility, considered by Hutten in [8]. However, in [8] physical possibility is identified with physical possibility$_2$, which in [3] is shown to have serious defects and to be fully unsatisfactory within foundations of the Mach-Painlevé type.

[4] The above word "possible", absent from the corresponding sentence in [3], can also be referred to genetic engineering — see [4, Ftn. 3].

Let us remark that in [3, pp. 200–201] a further explanation of physical possibilty$_1$ is given, mainly in classical physics; and besides avoiding some preceding counterfactual conditions, it affords *a criterion to (somehow) check* p'*s physical possibility experimentally.* In fact this explanation uses an arbitrary physical proposition p and a translation of it by the time τ (> 0), say $p' = p_\tau$; and it is based on the physical homogeneities of (classical) time and inertial spaces, as well as on the physical isotropy and mutual indistinguishability of these spaces if preferred.

Let us now consider the case where p and p' are referred to two space-time frames $\mathcal{F}$ and $\mathcal{F}'$ respectively (related to the same units of measure, possibly non-inertial, and) such that at all *corresponding* event-points $\mathcal{E} \in \mathcal{R}_4$ and $\mathcal{E}'$ $(\in \mathcal{R}_4)$ the gravitational, dragging, and Cariolis' forces per unit mass have the same representations in $\mathcal{F}$ and $\mathcal{F}'$ respectively.[5] Then the afore-mentioned criterion in [3] has this extended version:

A3.4 (a) For $\mathcal{R}_4$ bounded, p is (said to be) *physical possible*$_1$ if, for every approximation degree ε (> 0), some possible technicians — see Ftn. 4 — can implement some choice of $p' = p_\tau$ (for τ large enough) within the approximation degree ε.

(b) the analogue of A3.2 for physical possibility$_1$ holds.

While the explanations A3.1–3 (in effect in [3]) for the notions of causal and physical possibilities also work quite well in the space-times of both special and general relativity, the version A3.4 of the afore-mentioned criterion in [3] has been considered because it can be extended to the latter space-time rather naturally unlike the original version.

In fact we can consider any physical assertions p and p' that (i) express two conceivable phenomena occurring in the space-time regions $\mathcal{R}_4$ and $\mathcal{R}'_4$, respectively, of general relativity and (ii) are such that the descriptions of p, $\mathcal{R}_4$, and $\mathcal{R}_4$'s space-time metric in the former of some space-time frames $\mathcal{F}$ and $\mathcal{F}'$ (related to the same units of measure) equal the analogous description for p' in $\mathcal{F}'$. Now the following relativistic analogue of A3.4 can be stated.

A3.5 (a) For $\mathcal{R}_4$ bounded, p is (said to be) *physically possible*$_1$ if, for every event point $\overline{\mathcal{E}}$ of the real space-time Σ_ρ, and for every approximation degree ε (> 0), some future (possible) technicians — see Ftn. 4 — are able to implement some choice of the above p' together with the space-time metric of $\mathcal{R}'_4$, within the approximation degree ε and with $\mathcal{R}'_4$ belonging to $\overline{\mathcal{E}}$'s posterior causal cone (in Σ_ρ).

(b) Part (b) of A3.4 holds.

[5] That $\mathcal{E}$ and $\mathcal{E}'$ are *corresponding* event-points means here that they have the same coordinates in $\mathcal{F}$ and $\mathcal{F}'$ respectively.

The above criterion or explanation A3.5 is certainly more satisfactory than the one briefly written in [4, Ftn. 4, p. 55].[6] Incidentally the analogue of A3.5 for causal possibility and for p belonging to a possibly indeterministic theory also appears to hold.

Instead the relativistic explanation written in [4, Ftn. 4] risks giving too weak a meaning to "p's physical possibilty$_1$". The same can be said for the explanation (of the same kind and belonging to classical physics) written in [4, Ftn. 3], as well as for the one introduced in [3, p. 200] with the words, "Perhaps some readers prefer [...] the following [...]".[7]

4. As for the question raised in (B):

FIRST, I complete (ϑ) in sect. 2 by noting that in [1] I introduced my first formal framework and the first version of my axiomatization method[8] to render Painlevé's foundations [10] for classical particle mechanics rigorous (and to extend it). This was done because:

ι) a counterfactual conditional is (essentially) used in [10, p.65],

κ) the axioms effectively written in [10] are not sufficient to prove all theorems asserted there — see [1, p. 106]; e.g., [10] includes nothing similar to the possibility axiom (Post. 1) mentioned in [4, Ftn. 1] in order to sketch a definition of mass, or to the existence axiom for mass-distributions (implying certain necessity assertions) hinted at in η), and

λ) Painlevé seems aware that some improvements concerning his language (hence formal logic) and axiomatic system might be useful or necessary.[9]

[6] The (brief) relativistic explanation in [4, Ftn. 4] reads in effect as:

A3.6 A proposition p of any relativistic theory $\mathscr{T}$, that describes a (conceivable) phenomenon occurring in a space-time region $\mathcal{R}$ is said to be *physically possible*$_1$ if, for all $\mathcal{E} \in \mathcal{R}$, it is technically possible to implement a region $\mathcal{N}'$ isometric with some (bounded) neighborhood $\mathcal{N}$ of $\mathcal{E}$, together with the transform $p_{\mathcal{N},\mathcal{N}'}$ by the above isometry, of p's restriction $p_{\mathcal{N}}$ to $\mathcal{N}$ (with an arbitrarily preassigned approximation).

[7] Briefly speaking, as far as the explanation of "p's possibility$_1$" is concerned, the afore-mentioned risk of A3.6 in Ftn. 6 seems avoided by requiring p to describe a maximal conceivable evolution of the universe occurring in $\mathcal{R}$ (according to the physical theory being considered); and the same can be said of the afore-mentioned classical analogues of A3.6 in [4, Ftn. 3] and [3, p. 200].

[8] The formal framework presented in [1] consists of an unusual extensional language capable of dealing with modal notions. This framework is used in [4, sect. 4] in a generalized form.

[9] As is more thoroughly stated in [2, Ftns. 52–53, p. 110], in [10, pp. 64–65] Painlevé writes "Je voudrais enoncer rapidement ce corps d'axiomes" and, after his counterfactual conditional referred to in (ι), "cette terminologie admise les axiomes [...] ce resument ainsi."

μ) By (ι) and (λ) I was pushed to use mathematical logic — see Ftn. 8 — to solve the mathematical problems set in (κ), just as mathematics is used to solve physical problems. Later I wrote the more complex modal theory in [2] to interpret the physical language of Painlevé more directly, and especially to analyze some notions involved by axiomatizations of the Mach-Painlevé type better.

Incidentally I note that the above considerations, especially (ι), (λ), (μ) and Ftn. 9, serve to meet Urchs' request "for further explanation of how [my] proposal is related to what is really expressed in physical terminology by the notions [I exploit]" ([13, p. 41]) and that

ν) paper [11], belonging to the axiomatization of biology, as well as [5] explicitly refer to my second formal framework [2], the one set in [1] being insufficient for their purposes (modalities serve in [11] to render some definitions and laws rigorous).

Second, Painlevé's foundations [10] have been used in several textbooks of rational mechanics.[10] Furthermore — in compliance with (ϵ) and (η) (sect. 2) — it is very easy to check (κ), e.g. by reasoning similar to some essentiality considerations in [4, Ftn. 1] (see also [1, p. 106]). Lastly, essentially different rigorizations of [10] are not hinted at, as far as I know (see [4, Ftn. 2 (b)]).

By the First and Second points,

ξ) Being [1] a rigorous version of [10], it constitutes, so to speak, an indirect positive answer to the question in (B), and also an indirect "proof of reality" for Bressan's construction — see (B'') in sect. 1.

The following direct positive answers can be added. First, C. Truesdell, a well known mathematical physicist interested in axiomatizations admittedly not of the Mach-Painlevé type, shows a very favorable opinion regarding Bressan's axiomatic theory [1] — see [12, pp. 533–554] — while in [12, p. 532] he correctly refers to both [1] and [2] for other purposes.

Incidentally, in connection with (B') in sect. 1, it is interesting to note that Truesdell's considerations on [1] strongly agree with Garson's — see [7]; the latter is more interested in my interpreted formal framework, while the former explicitly notes that, e.g. I want to face "mathematical gaps", which is close to (μ).

Furthermore Truesdell contributed to and signed the presentation of Montanaro's paper labeled [193] in [4], which is in effect based on my formal system and involves some explicit modal axioms.

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[10] E.g. A. Signorini (Rome) and B. Finzi (Milan) followed [10], even if the latter regretted that thus some physical laws collapse into definitions.

Another direct positive answer to the question in (B) — and hence to the one on the "proof of reality" involved in (B'') — is given by (ν), in that [11] was written and presented to the press at the initiative of some biologists.

I conclude by briefly noting that, e.g., the rigorization [1] of Painlevé's foundations [10] renders these more complex; and a larger complexity arises when one starts an extension of [1] to continuous media.[11] However, even if this rigorization and extension decrease the possibility of divulgation, they have a scientific value, appearing from the preceding considerations. For example, even the aforementioned extension — implemented by, e.g. the papers [41], and [192] to [194] in [4] — can be useful to reach some concrete results, as it appears from the works [9] (a) and (b), as well as [6] — see Ftn. 1.

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[11] Foundations of the Mach-Painlevé type for continuous media are not at all superfluous, because their reduction to particle systems is troublesome, unlike the converse reduction, which has in effect been well known for nearly a century (see [12, pp. 537–538]).

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